The Fourth De Rham Curve
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Abstract

This article is a description of what has been attempted to be called the Bruhling curve, a new Lindenmayer system fractal which exhibits the properties of a De Rham curve.

Introduction

The Bruhling curve is a Lindenmayer system fractal curve. It is interesting because its ratio of growth is continuously non-differentable but may be defined exactly by a function of real numbers equal to $x(2)+2$, meaning the number of line segments double and add 2 to achieve the sum of line segments for the next iteration. This indicates diadic symmetry and that this fractal is the composition of a diadic monoid as it comprised of 2 discernable sides or elements. The fractal also exhibits self-similiarity meaning the overall shape is comprised of smaller, similiar shapes. These characteristics place the Bruhling curve within the definition of a De Rham curve. In this article we explore these characteristics through the transformations of this formula as well as the coding which reveals the recursive pattern of development along with what appears to be the location of the binary additive of $+2$ through each step. These associative binaries appear to define the beginning of either side of the monoid. Graphical renderings at varying angles of interest are also given to further support this theorem, and ultimately it is concluded that the Bruhling curve is of a De Rham type. Assuming this is true, it would make the Bruhling curve the 4th explored, with the Takagi, Cesaro-Levy, and Koch-Peano being the other 3 and would further our understanding and the definitions of this classification. This fractal was discovered by chance through the random writing and generation of Lindenmayer system formulas for my own recreation. Thus the following examination was constructed after this fractal was discovered and recognized to exhibit symmetry.

1. De Rham Curve

A De Rham curve is simply defined as a map where the point sets expand by a growth ratio that is infintely recursive and not rational at any and all steps but may be parameterized by a function of real numbers which remains constant throughout these steps. Below is the typical formula for a De Rham curve.

$$x = \sum_{k=1}^{\infty} \frac{b_k}{2^k}$$

This formula illustrates the expansion of point sets x as equal to the summation of each iterative step k, from step 1 through ∞, resulting of the function $\frac{b_k}{2^k}$, which is 2 times the summation, then the constant associative binary additive b plus the summation. This gives $d_x = d_{b_k}$, where d equals the dimensional or iterative step. If we replace the binary

additive b with the correlating binary additive value of the Bruhling curve, 2, we achieve the following,

$$x = \sum_{k=1}^{\infty} \frac{2_k}{2^k}$$

equaling $d_x = d_{2_k}$, giving the following transformations,

$$0(x) = 0(2) + 2 = 1(x)$$
$$1(x) = 2(2) + 2 = 2(x)$$
$$2(x) = 10(2) + 2 = 3(x)$$
$$3(x) = 22(2) + 2 = 4(x)....$$

and equaling the following summations. The ratio of growth for each iterative step is also given. It should be noted that the constant function of growth does not begin until after the 2nd iteration. The Bruhling curve requires the first two iterations to construct both sides of the monoid group and begin the constant function of growth.

Dimensional Step	Summation	Ratio of Growth
0	2	
1	4	4
2	10	2.5
3	22	2.2
4	46	2.090909090909090909
5	94	2.043478260869565217
6	190	2.021276595744680851
7	382	2.010526315789473684
10	3070	2.001303780964797914
15	98,302	2.000040691759918616
20	3,145,726	2.00000127156737209
25	100,951,294	2.000000039623068915

The summation of each iterative step is equal to the number of line segments for each corresponding iterative step of the Bruhling curve. This ratio of growth is non-differentiable, increasingly approaching, but > 2.

2. L-System formula

The Bruhling curve is expressed as a Lindenmayer system formula and may be written as follows,

Axiom FA
Theta 120 degrees
Rules $A = F + B - A$ $B = FA + FF + B$ $F = F$

The axiom is the seed code from which the 1ˢᵗ iteration is derived. It is the zero dimension and could be rendered graphically by generating at zero iterations. The rules define the values for the variables A, B as they're replaced through each iterative step. This value replacement is the definition of iteration. Since $F = F$, it could be considered a constant. The Theta defines the value of the angle of all turns as expressed by $+, -$. Below is an example of this variable replacement as expressed in the coding carried to the 6th iteration. . Shaded therein are what appear to be the location of the associative binary additives as they are created in alternates. These are the only places in the code where the variable FF+ occurs more than twice.

Dimensional Step	Code
0	FA
1	FF+B-A
2	FF+FA+FF+B-F+B-A
3	FF+FF+B-A+FF+FA+FF+B-F+FA+FF+B-F+B-A
4	FF+FF+FA+FF+B-F+B-A+FF+FF+B-A+FF+FA+FF+ B-F+FF+B-A+FF+FA+FF+B-F+FA+FF+B-F+B-A
5	FF+FF+FF+B-A+FF+FA+FF+B-F+FA+FF+B-F+B- A+FF+FF+FA+FF+B-F+B-A+FF+FF+B-A+FF+ FA+FF+B-F+FF+FA+FF+B-F+B-A+FF+FF+B- A+FF+FA+FF+B-F+FF+B-A+FF+FA+FF+B-F+ FA+FF+B-F+B-A
6	FF+FF+FF+FA+FF+B-F+B-A+FF+FF+B-A+FF+ FA+FF+B-F+FF+B-A+FF+FA+FF+B-F+FA+FF+ B-F+B-A+FF+FF+FF+B-A+FF+FA+FF+B-F+FA+ FF+B-F+A-B+FF+FF+FA+FF+B-F+B-A+FF+FF+ B-A+FF+FA+FF+B-F+FF+FF+B-A+FF+FA+FF+ B-F+FA+FF+B-F+B-A+FF+FF+FA+FF+B-F+B-A+ FF+FF+B-A+FF+FA+FF+B-F+FF+FA+FF+B-F+ B-A+FF+FF+B-A+FF+FA+FF+B-F+FF+B-A+FF+ FA+FF+B-F+FA+FF+B-F+B-A

The monoid group doubling observed here is a characteristic of a De Rham curve resulting from

the function $\dfrac{b_k}{2^k}$ as well as the the continuous non-differentiability found throughout the coding.

The self-recursive nature of the coding may be observed here as well, through the replication of code patterns though may be more easily viewed as self-similiarity in graphical renedrings.

3. Renderings

The following are graphical renderings of the Bruhling curve formula viewed at the angles of 120, 90, 60 and 30 degrees.

First, 120 degrees, iterations 1-16, which gives a C-curve similar to the Cesaro-Levy curve, though exhibiting a triangular and pentagonal patterning. And, Unlike the Cesaro-Levy curve, the largest pattern formation on the Bruhling curve transitions from center to the adjacent sides and back going through the increasing iterative steps. It should be noted that this could coinside with
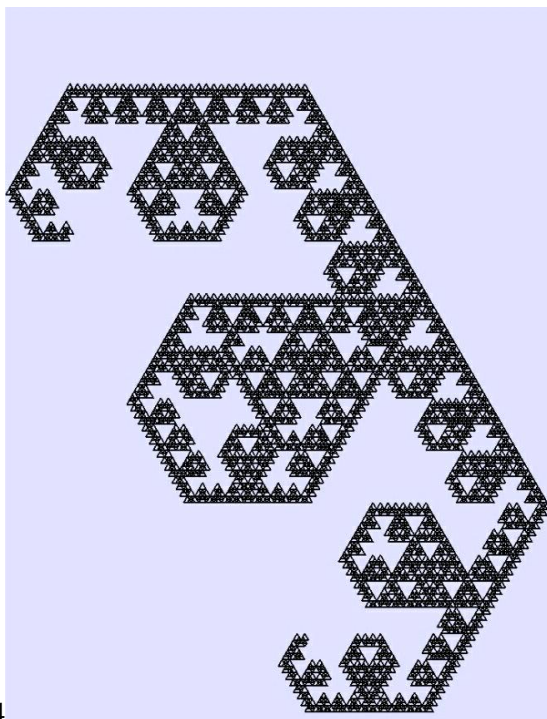
the alternation of the placement of the associative binary additives within the coding as they are introduced with each iteration.

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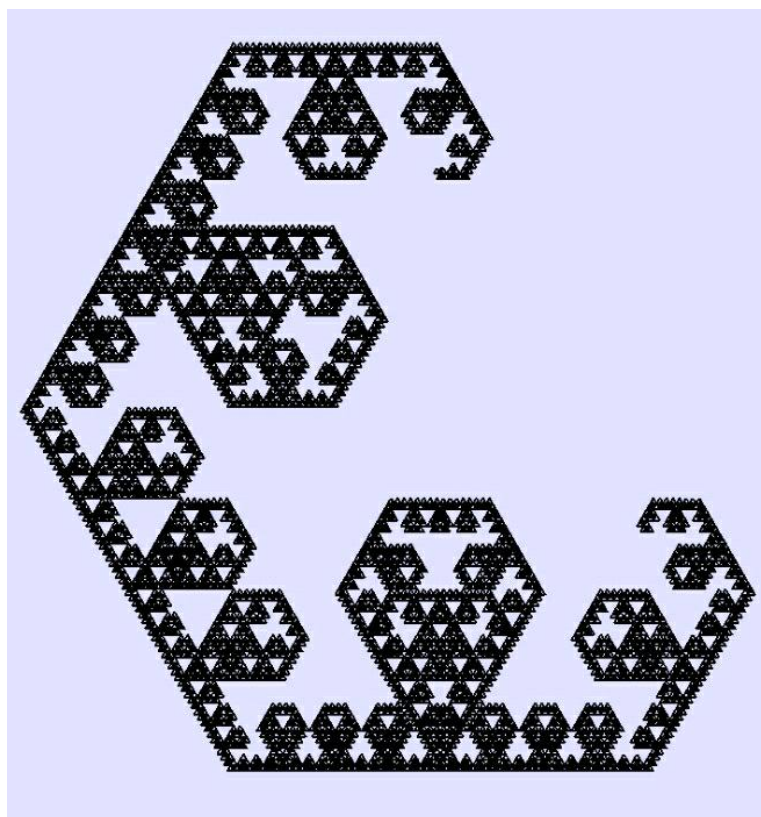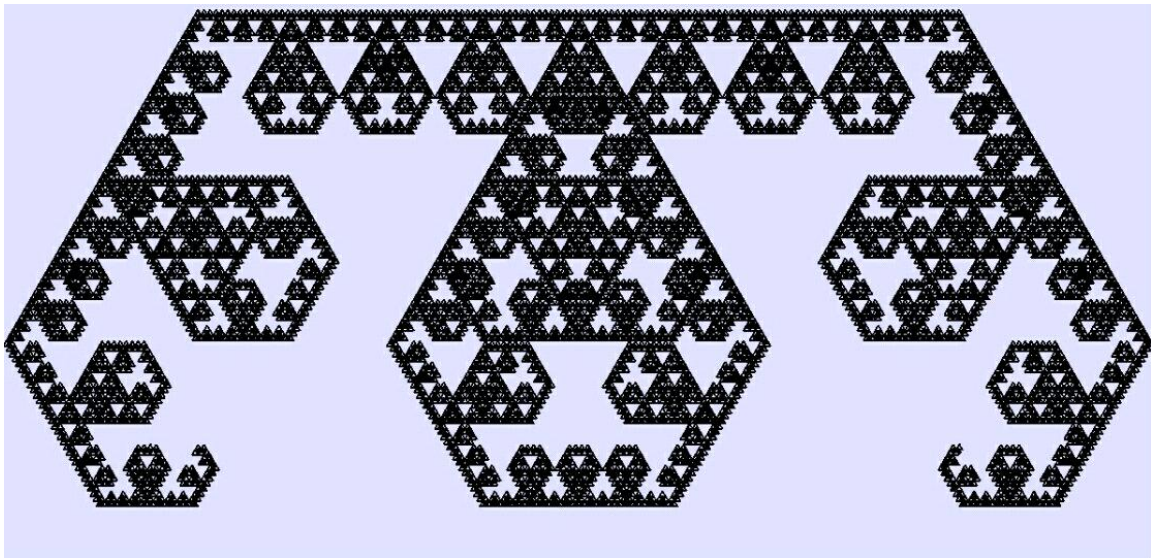2

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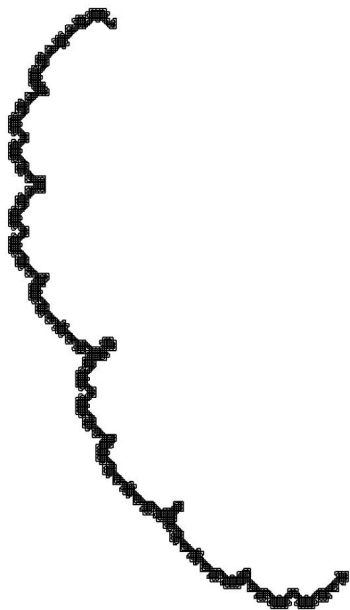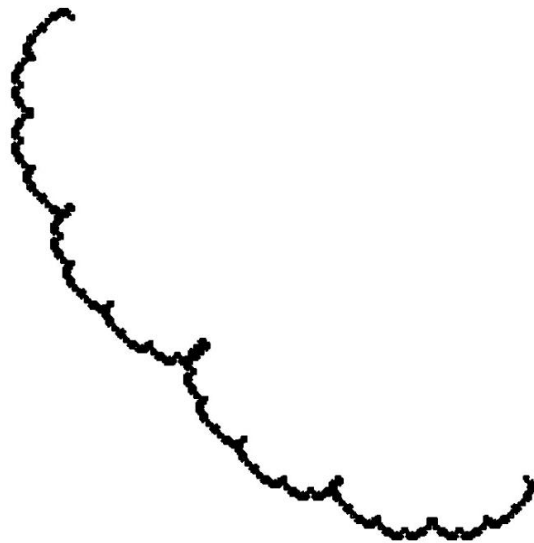
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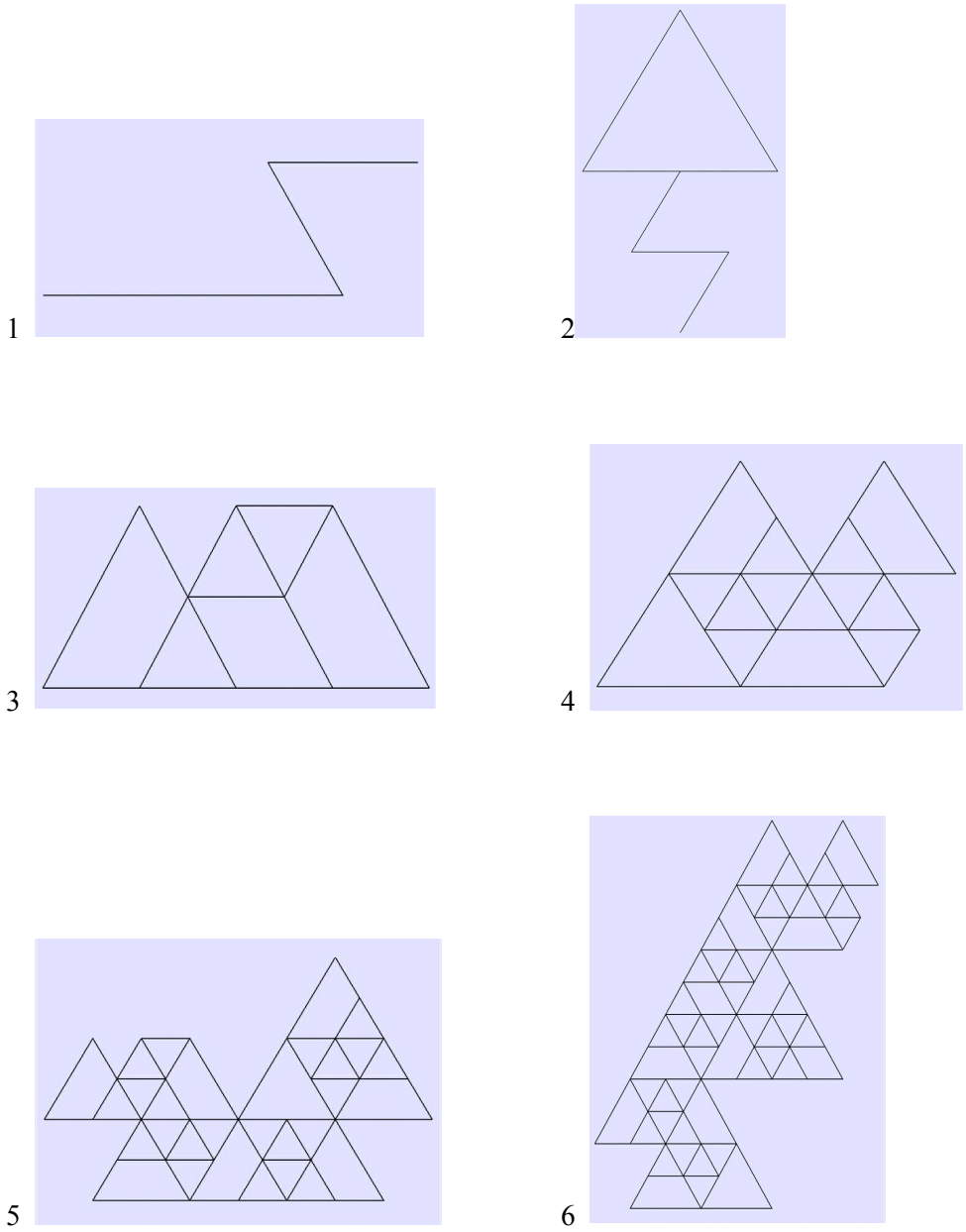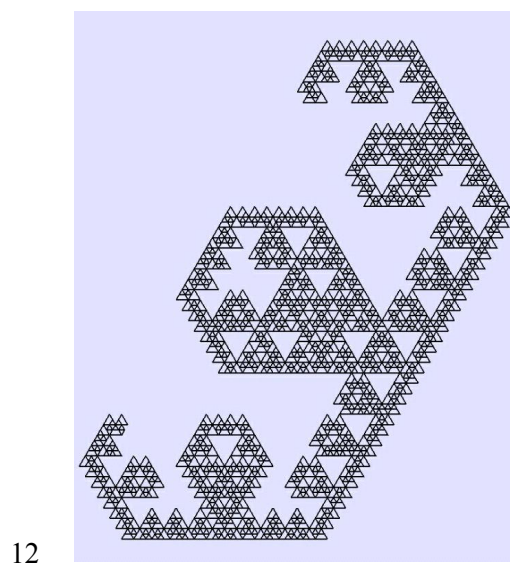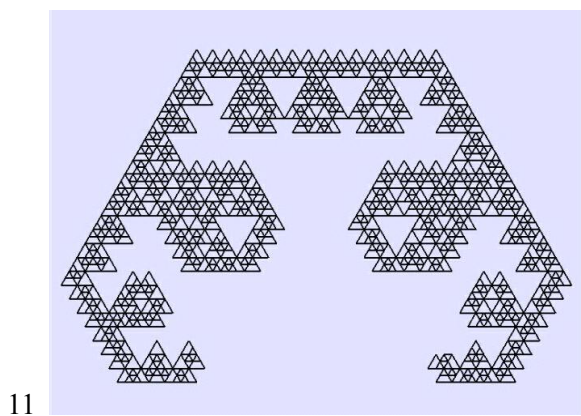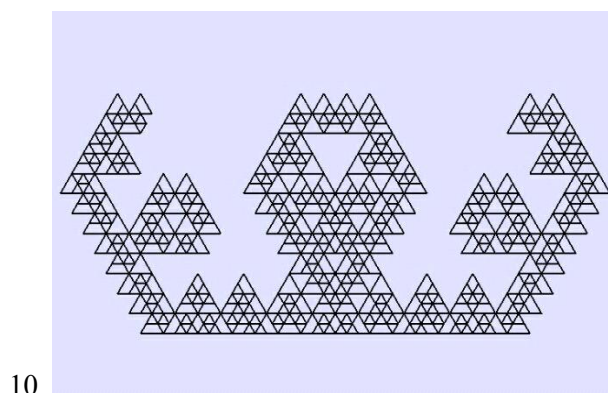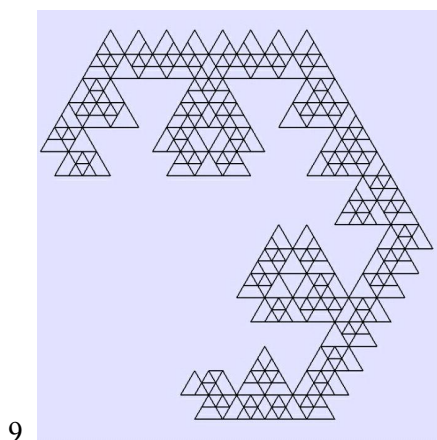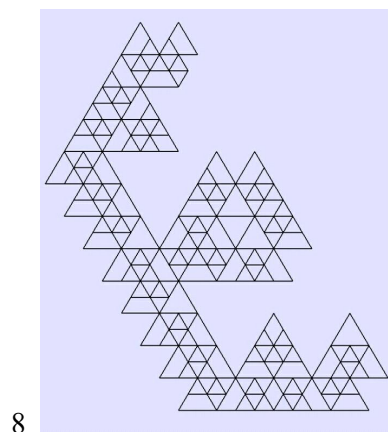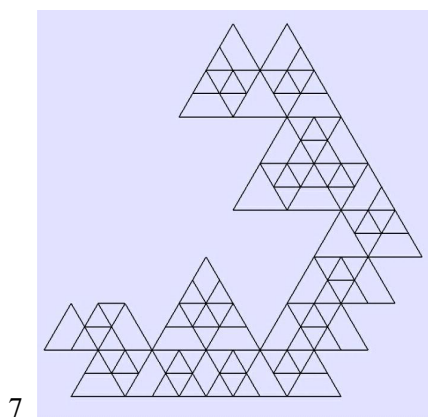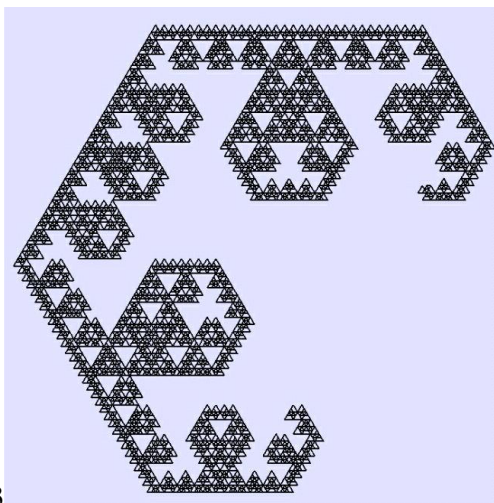
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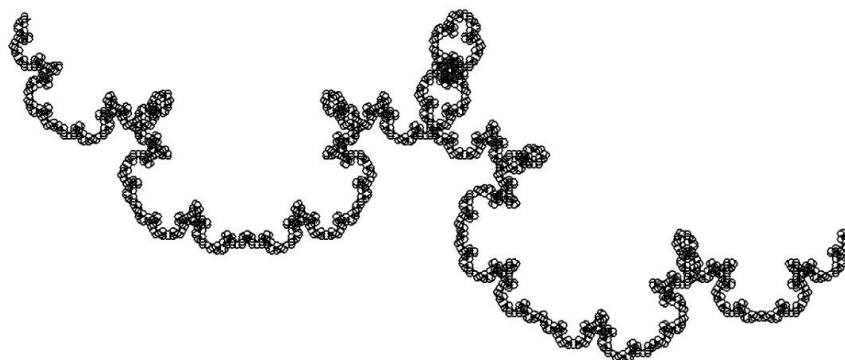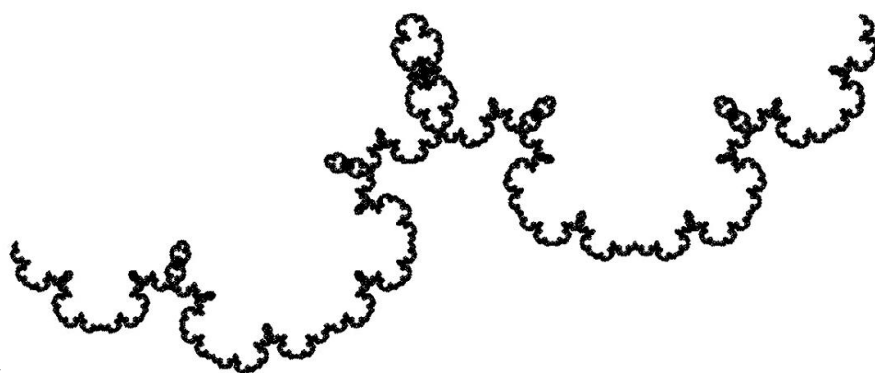16

Next, 90 degrees, iterations 13-16.
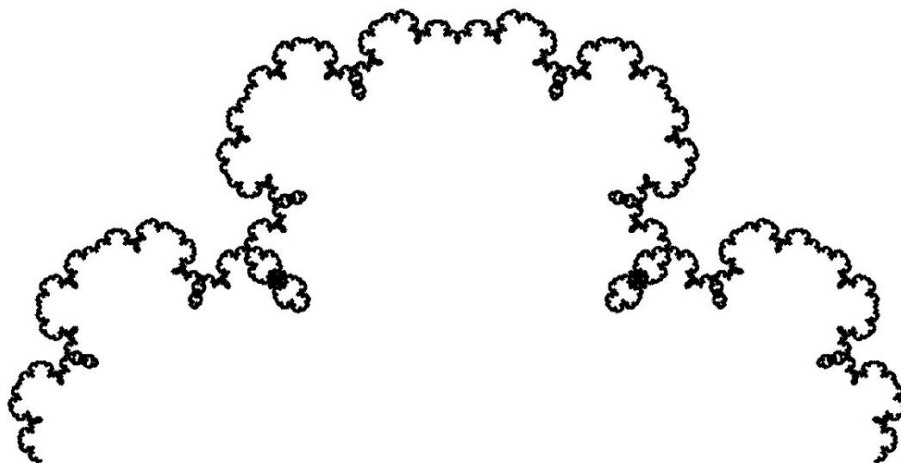

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Next 60 degrees, iterations 13-16.
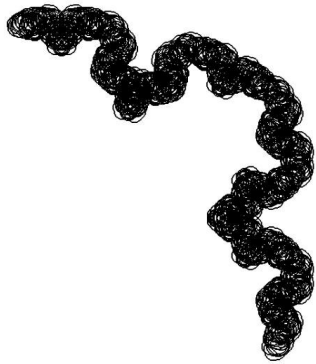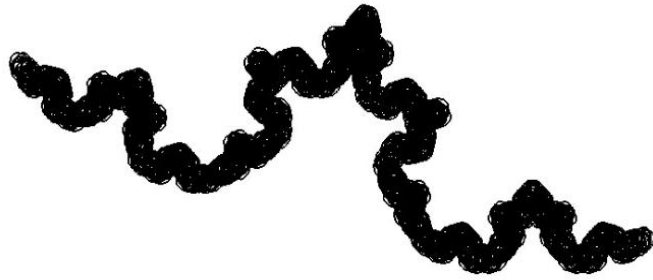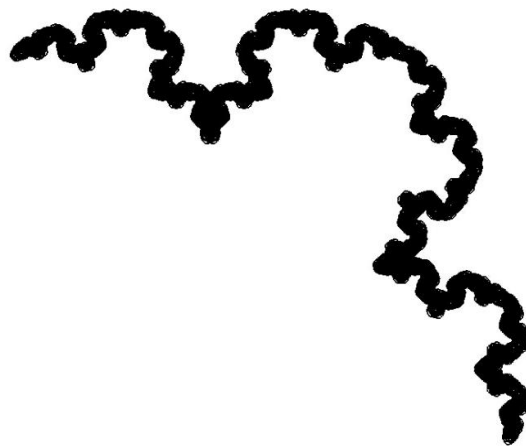

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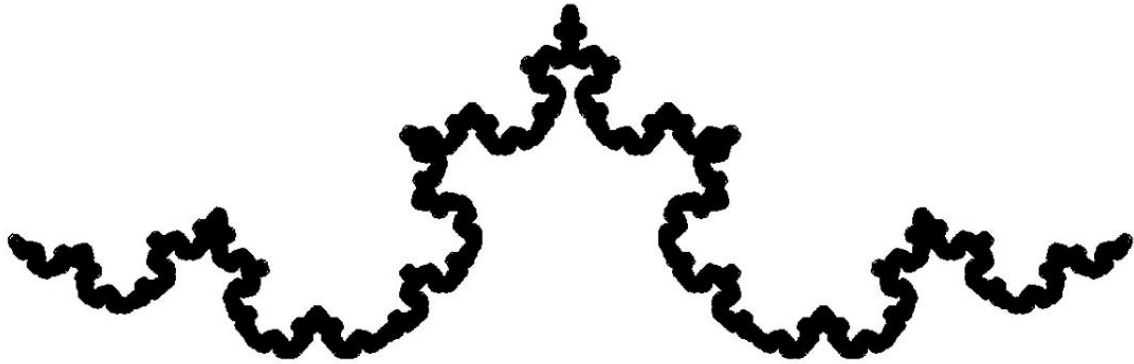Lastly 30 degrees, iterations 13-16.

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In conclusion, the statements provided here supporting the theorem postulated that the Bruhling curve is of a De Rham type, as its characteristics are definitive of a De Rham curve, appear sound and reasonable, and are correct and true to the best of my knowledge. Below are references that were used to help derive this conclusion and construct this examination. Fractal images were rendered using "Aristid" L-system generator.

1. Linas Vepstas -A Gallery of de Rham Curves-   pg. 1-7, 08/20/06

2. Bernt Wahl -Fractal Explorer, Chapter 4 Calculating: Fractals Dimensions-
     http://www.wahl.org/fe/HTML_version/link/FE4W/c4.htm

3. -Wikipedia articles- De Rham curve, Dragon curve, Hausdorff Dimension, Levy C curve,
     monoid, symmetry